Krishna University Pre-PhD Examination
Syllabus for Mathematics

PAPER- I
RESEARCH METHODOLOGY

UNIT-I

A) **Introduction** : Meaning of Research, Objectives of Research, Motivation in Research, Types of Research, Research Approaches, Significance of Research, Research methods Versus methodology, Research and Scientific Method, Importance of Knowing How Research is Done, Research process, Criteria of good Research

B) **Research Problem :** What is a Research Problem, Selecting the Problem, Necessity of defining the problem, Technique involved in defining a problem, An illustration, Conclusion.

(Chapters 1 and 2 of [1])

UNIT-II

A) **Reviewing the Literature:** Place of literature review in research, Bring clarity and focus to research problem,Improve methodology, Broaden Knowledge base in research area, contextualise findings, procedure for reviewing the literature, Search for existing literature, review the literature selected, develop a theoretical framework, develop a conceptual framework, writing up the literature reviewed. (Chapter 3 of [2])

B) **Research Design:** Meaning of research design, need for research design, features of a good design, Important concepts relating to research design, Different research designs. (Chapter 3 of [1])

UNIT-III

A) **Methods of Data Collection:** Collection of primary data, Observation method, Interview method, Collection of Data through questionnaires, Collection of data through schedules, Difference between questionnaires and schedules, Some other methods of data collection, Collection of Secondary data, Selection of Appropriate method for data collection. (Chapter 6 of [1])

B) **Processing and Analysis of Data**: Processing operations, Some Problems in processing, Elements/types od Analysis, Statistics in research, Measures of central tendancy,dispersion, Asymmetry and relationship., Simple regression analysis, Multiple correlation and regression, Partial correlation. (chapter 7 of [1]).

UNIT-IV

A) **Mathematical Logic and Proof:** Logic, Propositional Equivalences, Predicates and Quantifiers, Nested Quantifiers, Methods of proof.

B) **Mathematical Reasoning, Induction and Recursion :** Proof strategy**,** Sequences and Summations, Mathematical Induction, Recursive Definitions and Structural Induction.

(Chapters 1 and 3 of [3])

UNIT-V

A) **Interpretation and Report Writing :** Meaning of Interpretation, Why Interpretation, Technique of Interpretation, Precaution in Interpretation, Significance of Report Writing, Different steps in Writing Report, Lay out of Research Report, Types of Reports. (Chapter 14 of [1]).

B) **Report Writing** : Introduction, Pre writing considerations: Dissertations/ Theses, Style and composition of the Report, Principles of Thesis Writing, Format of Reporting in Dissertations, Research reports , Publications in a Research Journal, Reporting of Qualitative Research, Briefing, Rules for typing or Word Processing. (Chapter 19 of [4]).

Prescribed books :

[1] C.R.Kothari, " Research Methodology – Methods and Techniques" , Second Revised Edition, New Age International Publisheres.

[2] Ranjit Kumar, " Research Methodology- A step by step guide for beginners", Second Edition, Pearson Education (Singapore) Pte. Ltd.

[3] Kenneth H. Rosen, "Discrete Mathematics and its Applications". Fifth Edition, Tata McGraw – Hill Publishing Company Limited.

[4] K.N.Krishna Swamy, Appa Iyer Siva Kumar, M.Mathirajan, " Management Research Methodology", Pearson.

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PAPER- II

UNIT-I

C) Group theory: Definition of a Group - Some Examples of Groups - Some preliminary Lemmas - Subgroups - A Counting Principle - Normal Subgroups and Quotient Groups - Homomorphisms.

B) Ring Theory: Definitions and Examples of Rings - some special classes of rings - Homomorphisms - Ideals and quotient Rings - More Ideals and quotient Rings.

Prescribed book: I.N.Herstien, "Topics in Algebra", Wiley Eastern Ltd., New Delhi,

UNIT-II

Multivariable Differential Calculus

C) Introduction - The directional derivative- Directional derivatives and Continuity - The total derivative- The total derivative expressed in terms of partial derivatives - An Application to complex valued functions -The matrix of a linear function - The Jacobian matrix- The chain rule - Matrix form of the chain rule.

D) The mean-value theorem for differentiable functions - A sufficient condition for differentiability - A sufficient condition for equality of mixed partial derivatives- Taylor's formula for functions from R^n to R^1 .

Prescribed book : TOM.M.APOSTOL, "Mathematical Analysis" ,Second Edition Narosa publishing Company.

UNIT-III

C) Analytic and Harmonic functions: Differentiable and Analytic functions - The Cauchy Riemann equations – Harmonic functions

Complex integration: Complex Integrals –Contours and Contour Integrals - The Cauchy Goursat theorem – The fundamental Theorems of Integration -Integral representations for analytic functions, The Theorems of Morera and Liouville, and Extensions

D) Residue theory: The Residue theorem -Trigonometric Integrals – Improper Integrals of rational functions – Improper Integrals involving trigonometric functions – Indented contour integrals.

Prescribed book: John H. Mathews and Russel W, Howell, "Complex Analysis for Mathematics and Engineering",Fifth Edition, Narosa publishing house

UNIT-IV

A)Numerical techniques for solving ordinary differential equations: Introduction- Numerical method- Single step methods- Multi step methods- Predictor Corrector methods. (Sections 6.1 to 6.5 of chapter 6 of [1])

B)Numerical methods for solving elliptic partial differential equations:Introduction- Difference methods- Introduction- Difference methods for Linear boundary value Problems. (Sections 1.1, 1.2, 4.1 to 4.2 of [2]).

Prescribed books:

[1]. Numerical methods for Scientific and Engineering Computation, M.K.Jain, S.R.K. Iyengar and R.K. Jain, 3rd edition, 1993, New Age International Pvt.Ltd.

[2]. Computational methods for partial differential equations by M.K. Jain, S.R.K.Iyengar and R.K. Jain, New Age International Pvt. Ltd. (1993).

UNIT-V

C) The Laplace Transform: Definition, Existence and basic properties of the Laplace transform – The Inverse Transform and the Convoltution- Laplace transform solution of linear differential Equations with constant coefficients.

D) Sturn Liouville Boundary value problems and Fourier Series: Sturn Liouville Problems – Orthogonality of Characteristic functions- The expansion of function in a series of orthonormal functions – Trigonometric Fourier Series

Prescribed book: Shepley .L.Ross “ Differential Equations”, Third Edition , John Wiley & Sons

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PAPER- III

UNIT-I

A) Ring Theory: The field of quotients of an Integral domain- Euclidean Rings- A Particular Euclidean Ring- Polynomial Rings (from [1])

B) Modules: Definition and Examples- Submodules and direct sums- R homomorphisms and Quotient Modules- Completely Reducible modules (from [2])

Prescribed books:

[1] I.N.Herstien, 'Topics in Algebra', Wiley Eastern Ltd., New Delhi

[2] P.B.Bhattachrya, S.K.Jain, S.R.Nagpaul, 'Basic Abstract Algebra'**,** Cambridge University Press

UNIT-II

E) Vector Spaces: Elementary Basic Concepts- Linear Independence and Bases – Dual Spaces (from [1])

F) Algebraic Extensions of Fields: Irreducible Polynomials and Eisenstein Criterion- Adjunction of roots- Algebraic extensions.(from [2])

Prescribed books:

[1] I.N.Herstien, Topics in Algebra, Wiley Eastern Ltd., New Delhi

[2] P.B.Bhattachrya, S.K.Jain, S.R.Nagpaul, 'Basic Abstract Algebra'**,** Cambridge University Press

UNIT-III

E) Power Series Solutions of Ordinary Differential Equations: Introduction- A review of Power Series- Series Solutions of First order equations- Second order Linear Equations- Ordinary Points- Regular Singular Points- Regular Singular Points(Continued)

F) Special Functions: Legendre Polynomials- Properties of Legendre polynomials- Bessel Functions- The Gamma Function- Properties of Bessel Functions

Prescribed book:

George F.Simmons **' Differential Equations with Applications and Historical Note**s' 2nd Edition, Tata Mc Graw Hill publishing Company Ltd.

UNIT-IV

A) **Fundamental Concepts of Graph Theory:** What is a Graph – Definition-Graphs as Models – Matrices and Isomorphism- Decomposition and Special Graphs- Connection in Graphs- Bipartite Graphs- Eulerian Circuits
B) **Trees and Distance:** Properties of Trees – Distance in trees and Graphs- Enumaration of Trees- Spanning trees in Graphs- Decomposition and Graceful Labelings.

Prescribed book: Douglas B.West , 'Introduction to Graph Theory', PHI, 2003

UNIT-V

E) **Kinematics of Fluids in Motion**: Real Fluids and Ideal Fluids- Velocity of Fluid at a Ponit- Stream Lines and Path lines- Steady and unsteady flows- the velocity potential- The vorticity Vector- Local and Particle rates of change- The equation of Continuity – Examples.
F) **Equations of motion of a fluid:** Pressure at a point in a fluid at rest- Pressure at a point in a moving fluid- conditions at a boundary of two inviscid immiscible fluids- Euler's equations of motion- Bernoulli's Equation –Examples.

Prescribed book: Frank Chorlton, 'Text book of Fluid Dynamics' CBS publications

Krishna University, Machilipatnam
M.Phil / Pre-PhD Mathematics Examination
Model Question Paper
PAPER- I
RESEARCH METHODOLOGY

Time: 3 hours | **Answer ONE question form each unit.** | **Max. Marks: 100**

All questions carry equal marks.

UNIT – I

1. What do you mean by research? Describe the different steps involved in a research process.

(OR)

2. How do you define a research problem? Describe the techniques of defining a research problem with an example.

UNIT – II

3. Explain the procedure for reviewing the literature.

(OR)

4. Explain the meaning and significance of a research design.

UNIT – III

5. Enumerate the different methods of collecting data. Explain the difference between collection of data through questionnaires and schedules.

(OR)

6. " Processing of data implies editing, coding, classification and tabulation ". Describe in brief these four operations pointing out the significance of each in context of research study.

UNIT – IV

7. (a) Show that the following implication is a tautology by using truth table
$$\left[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)\right] \rightarrow r .$$
(b) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

(OR)

8. (a) Explain mathematical induction. Use mathematical induction to show that
$$\sum_{j=0}^{n} ar^{j} = \frac{ar^{n+1} - a}{r - 1}$$, when $r \neq 1$.
(b) Explain recursion. Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$, where $\alpha = (1+\sqrt{5})/2$ and $f_0=0$, $f_1=1$, $f_n= f_{n-1}+f_{n-2}$ for n=2,3,4,..... are the Fibonacci numbers.

UNIT – V

9. (a) Write a brief note on the " Task of interpretation" in the context of research methodology.
(b) Explain the significance of a research report and narrate the steps involved in writing such a report.

(OR)

10. (a) Explain the principles of thesis writing.
    (b) Explain the format of thesis/ Dissertation.

Krishna University, Machilipatnam
M.Phil / Pre-PhD Mathematics Examination
Model Question Paper
PAPER- II

Time: 3 hours **Answer ONE question form each unit.** **Max. Marks: 100**
All questions carry equal marks.

UNIT – I

1. (a) Define a group. If H is a finite subset of a group G and H is closed under multiplication, then prove that H is a sub group of G.
   (b) State and prove Cauchy's theorem for abelian groups.

(OR)

2. (a) Prove that a finite integral domain is a field.
   (b) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

UNIT – II

3. (a) Define the total derivative of a function from R^n in to R^m. Let u and v be two real valued functions defined on a subset S of the complex plane. Assume that u and v are differentiable at an interior point c of S and that the partial derivatives satisfy the Cauchy – Riemann equation at c. Then show that the function f = u+ iv has a derivative at c and $f^I(c) = D_1u(c)+i\,D_2v(c)$.
   (b) State and prove chain rule for differentiation.

(OR)

4. (a) Assume that one of the partial derivatives $D_1f, D_2f, \ldots, D_nf$ exists at c and that the remaining (n-1) partial derivatives exist in some n – ball B(c) and are continuous at c. Then show that f is differentiable at c.
   (b) State and prove Taylor's formula for functions from R^n to R^I.

UNIT – III

5. (a) Let f(z) = u(x,y)+i v(x,y) be an analytic function on a domain D. Then show that both u and v are harmonic functions on D.

   (b) State and prove Cauchy-Goursat theorem and evaluate $\int_c \frac{dz}{(z-z_0)^n}$, where n>1

   and c is a simple closed continuous curve with positive orientation such that z_0

   lies interior to c.

(OR)

6. (a) Define residue of a function . State and prove Cauchy's Residue theorem, and evaluate

$$\int_0^{2\pi} \frac{\cos 2\theta}{5-4\cos\theta} d\theta .$$

(b) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^3}$.

UNIT – IV

7. (a)Explain Euler method for solving the Initial value problem $u^I = f(u)$ with initial condition $u(f_0) = u_0$.
   (b) Solve the Initial value problem $y^I = 1+y^2$, with initial conditions y=0 when x=0 and find y(0.2), y(0.4) using fourth order Runge- Kutta method.

(OR)

8. (a) Classify the partial differential equation $\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial t \partial x} + 4\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$ and find its characteristics. Reduce the equation to its standard form.
   (b)Solve the mixed boundary value problem

$$\nabla^2 u = 0, \quad 0 \le x, y \le 1$$

$$\left.\begin{array}{ll} u = 2x, & y=0 \\ u = 2x-1, & y=1 \end{array}\right\} \quad 0 \le x \le 1$$

$$\left.\begin{array}{ll} u_x + u = 2-y, & x=0 \\ u = 2-y, & x=1 \end{array}\right\} \quad 0 \le y \le 1$$

Use the five point formula with h = k= 1/3.

UNIT – V

9. (a) State and prove Existence theorem for Laplace transformations.
   (b) Use Laplace transformations to solve the Initial value problem

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 2y = 10\cos t, \quad y(0)=0,\ y^I(0)=0,\ y^{II}(0)=3.$$

(OR)

10. (a) Define a Sturm – Liouville problem. Find the characteristic values and characteristic functions of the Sturm – Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0$, y(0)=0, y(π/2)=0.
    (b) Define the trigonometric Fourier series of f on the interval –L ≤ x≤ L . Find the trigonometric Fourier series of the function f defined by f(x) = x, -4 ≤x≤ 4, on the interval -4 ≤x≤ 4.

Krishna University, Machilipatnam
M.Phil / Pre-PhD Mathematics Examination
Model Question Paper
PAPER- III

Time: 3 hours | **Answer ONE question form each unit.** | **Max. Marks: 100**
All questions carry equal marks.

UNIT – I

11. (a) Define an Euclidean ring . Prove that any Euclidean ring is a principal ideal ring.
(b) Prove that an Ideal A = (p(x)) in F[x] is a maximal ideal if and only if p(x) is irreducible over F.

(OR)

12. (a) Define a finitely generated R – module. Show that if an R- module M is generated by a set $\{x_1, x_2, ..., x_n\}$ and $1 \in R$ then $M = \{r_1x_1+r_2x_2+....+ r_nx_n / r_i \in R\}$.
(b) Let R be a ring with unity and let M be an R – Module. Then show that the following are equivalent:
    (i) M is simple
    (ii) $M \neq (0)$ and M is generated by any $0 \neq x \in M$
    (iii) $M \cong R/I$, where I is a Maximal left ideal of R.

UNIT – II

13. (a) Define a vector space. If V is a finite dimensional vector space and W is a subspace of V, then show that W is finite dimensional, $\dim W \leq \dim V$ and dim (V/W)= dim V- dim W.
(b) If V is the internal direct sum of its sub spaces $U_1, U_2, ...U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, ...U_n$.

(OR)

14. (a) State and prove Eisenstein criterion.
(b) Let E be an extension field of F and let $u \in E$ be algebraic over F. Let $p(x) \in F[x]$ be a polynomial of least degree such that p(u)=0. Then prove that
    (i) p(x) is irreducible over F.
    (ii) If $g(x) \in F[x]$ is such that g(u) =0, then p(x)|g(x) .
    (iii) There is exactly one monic polynomial $p(x) \in F[x]$ of least degree such that p(u)=0.

UNIT – III

15. (a) Find the general solution of $(1+x^2)y'' + 2xy' – 2y = 0$ in terms of power series in x.
(b) The equation $4x^2y'' – 8x^2y' + (4x^2+1)y = 0$ has only one Frobenius series solution. Find its general solution.

(OR)

16. (a) Find the first three terms of the Legendre series of $f(x) = \begin{cases} 0 & \text{if } -1 \leq x \leq 0 \\ X & \text{if } 0 \leq x \leq 1 \end{cases}$

(b) Prove that $\int_0^1 xJ_p(\lambda_m x)J_p(\lambda_n x)dx = \begin{cases} 0 & \text{if } m \neq n, \end{cases}$

$$\frac{1}{2} J_{p+1}(\lambda_n)^2 \quad \text{if m=n,}$$

where $J_p(x)$ is the Bessel function of first kind of order p and λ_n are the positive zeros of $J_p(x)$.

UNIT – IV

17. (a) Define a bipartite graph. Show that the complete graph K_n can be expressed as the union of k bipartite graphs if and only if $n \le 2^k$.

    (b) Define an Eulerian graph. Show that a graph G is Eulerian if and only if it has at most one non trivial component and its vertices all have even degree.

(OR)

18. (a) Define a tree. Show that the centre of a tree is a vertex of an edge.

    (b) Define a graceful labeling of a graph. If a tree T with m edges has a graceful labling, then prove that K_{2m+1} has a decomposition in to 2m+1 copies of T.

UNIT – V

19. (a) Derive the equation of continuity for the flow of a fluid.

    (b) Test whether the motion specified by $q = \frac{k^2(xj - yi)}{x^2 + y^2}$ (k = constant) is a possible motion for an incompressible fluid. If so, determine the equations of the streamlines. Also test whether the motion is of the potential kind and if so, determine the velocity potential.

(OR)

20. (a) Derive the Euler's equation of motion.

    (b) A long pipe is of length *l* and has slowly tapering cross-section. It is inclined at an angle α to the horizontal and water flows steadily through it from the upper to the lower end. The section at the upper end has twice the radius of the lower end. At the lower end, the water is delivered at atmospheric pressure. If the pressure at the upper end is twice atmospheric pressure, then find the velocity of delivery.