MAT 401 – NON COMMUTATIVE RINGS

UNIT I

Classical theory of Associative rings: Primitive Rings, Radicals, Completely reducible modules.
[Sections 3.1, 3.2, 3.3 of Chapter 3]

UNIT II

Classical theory of Associative rings: Completely reducible rings, Artinian and Noetherian rings,
[Sections 3.4, 3.5 of Chapter 3]

UNIT III

Classical theory of Associative rings: On lifting idempotents, Local and Semi perfect rings.
[Sections 3.6, 3.7 of Chapter 3]

UNIT IV

Injectivity and Related concepts: Projective modules, Injective modules
[Sections 4.1, 4.2 of Chapter 4]

UNIT V

Injectivity and Related concepts: The complete ring of quotients, Rings of endomorphisms of
Injective modules.
[Sections 4.3, 4.4 of Chapter 4]

PRESCRIBED BOOK:  J. Lambek, Lectures on Rings and Modules, A Blasidell book in
Pure and Applied Mathematics.

REFERENCE BOOK:  Thomas W. Hungerford, Algebra, Springer publications
M.Sc. DEGREE EXAMINATION
MODEL QUESTION PAPER
Fourth Semester
Mathematics
Paper I – NON COMMUTATIVE RINGS

Time: Three hours
Answer ONE question from each unit. Maximum: 70 Marks
All questions carry equal marks.

UNIT I

1. (a) Define a primitive ring. Show that the ring $R$ is primitive if and only if there exists a faithful irreducible module $A_R$.

   (b) Define a prime radical. Show that the prime radical of a ring $R$ consists of all strongly nilpotent elements.

   (OR)

2. (a) Show that the prime radical of a ring $R$ is the smallest ideal $K$ such that $R/K$ is semi prime.

   (b) Define a completely reducible module, socle of a module. Show that the following conditions concerning a module $A$ are equivalent.

   (i) $A$ is completely reducible.
   (ii) $A$ has no proper large sub module.
   (iii) $L(A)$ is complemented.

UNIT II

3. (a) If $K$ is any minimal right ideal of a ring $R$, then show that either $K^2 = 0$ or $K = e R$, where $e^2 = e \in K$.

   (b) Define a semi prime ring. If $R$ is semi prime and $e^2 = e \in R$ then show that $eR$ is a minimal right ideal if and only if $eRe$ is a division ring.

   (OR)

4. State and prove Hilbert- basis theorem.

UNIT III

5. (a) Let $N$ be a nil ideal of a ring $R$. Then show that idempotents modulo $N$ can be lifted.

   (b) Define a semi perfect ring. Show that any semi perfect ring contains a finite orthogonal set of primitive idempotents whose sum is 1.
6 (a) Define a local ring. Show that if a ring R is semi perfect and e is a primitive idempotent of R, then show that eRe is local.

    (b) Let R be semi perfect and R/ Rad R is prime then show that R is isomorphic with the ring of all endomorphisms of a finitely generated free S- module , where S is a local ring.

UNIT IV

7 (a) Define a projective module. Show that every R-module is projective if and only if R is completely reducible.

    (b) show that every R- module is projective if and only if R is completely reducible.

(OR)

8 (a) Define an Injective module. Prove that a module M is injective if and only if M has no proper essential extension.

    (b) Define a Divisible group. Prove that an abelian group is injective if and only if it is divisible

UNIT V

9 (a) Define complete ring of quotients. Show that if D is a dense right ideal of a ring R then show that for any q ∈ Q, q⁻¹D = {r ∈ R/ qr ∈ D} is also dense.

    (b) Let R be a prime ring with non zero socle .Then show that Q is the ring of all linear transformations of a vector space.

(OR)

10 . Let Mₖ be finite dimensional module. Then show that

(a) its injective hull Iₖ is the direct sum of a finite number of indecomposable injective modules .

(b) The ring H of endomorphisms of Iₖ is semi perfect .

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(MAT 401)
KRISHNA UNIVERSITY
M.SC MATHEMATICS SYLLABUS
(with effect from the batch admitted during 2012-13)

IV SEMESTER

MAT 402 – MEASURE AND INTEGRATION

UNIT-I

Lebesgue Measure: Introduction, Outer measure, Measurable sets and
Lebesgue measure, A nonmeasurable set, Measurable functions, Littlewood’s three
principles (Chapter 3)

UNIT-II

The Lebesgue Integral: The Riemann Integral, The Lebesgue Integral of a
bounded function over a set of finite measure, The Integral of a non-negative function,
The general Lebesgue Integral. (Sections 4.1 to 4.4 of Chapter 4).

UNIT-III

Differentiation and Integration: Differentiation of monotone functions, Functions of
bounded variation, Differentiation of an Integral, Absolute continuity.
(Sections 5.1 to 5.4 of Chapter 5)

UNIT-IV

Measure and Integration: Measure spaces, Measurable functions, Integration,
General Convergence theorems, Signed Measures, The Radon-Nikodym theorem.
(Sections 11.1 to 11.6 of Chapter 11)

UNIT-V

Measure and Outer Measure: Outer Measure and Measurability, The Extension
theorem, Product measures.
(Sections 12.1,12.2 & 12.4 of Chapter 12).


REFERENCE BOOKS: [1] P.R.Halmos, Measure Theory, Springer-Verlag, 1974
M.Sc. DEGREE EXAMINATION
MODEL QUESTION PAPER
Fourth Semester - Mathematics
Paper II – MEASURE AND INTEGRATION

Time: Three hours
Answer ONE question form each unit.
Maximum: 70 Marks
All questions carry equal marks.

UNIT I

1. Define Outer measure and Show that the Outer measure of an Interval is its length
   (OR)

2. (a) State and Prove Egoroff’s theorem.
   (b) If \( \{E_n\} \) is a decreasing sequence of measurable sets with \( m(E_1) \) finite, then show that
   \( m(\bigcap E_n) = \lim n m(E_n) \).

UNIT II

3 (a) State and Prove Fatou’s Lemma.
   (b) State and Prove bounded convergence theorem.
   (OR)

4 (a) Let \( f \) be a non-negative function which is integrable over a set \( E \). Then Show that given \( \epsilon > 0 \), there
   is a \( \delta > 0 \) such that for every set \( A \subset E \) with \( m(A) < \delta \),
   \( \int_A f < \epsilon \).
   (b) State and Prove Lebesgue Convergence Theorem.

UNIT III

5 State and Prove Vitali Covering Lemma.
   (OR)

6 (a) If \( f \) is absolutely continuous on \([a,b]\) and \( f'(x) = 0 \) a.e., then show that \( f \) is constant.
   (b) If \( f \) is bounded and measurable on \([a,b]\) and \( F(x) = \int_a^x f(t) \) \( + F(a) \), then show that \( F'(x) = f(x) \) for
   almost all \( x \) in \([a,b]\).

UNIT IV

7 (a) Define Positive set and Negative set with respect to a signed measure \( \nu \). Let \( E \) be a
   measurable set such that \( 0 < \nu(E) < \infty \), then show that there is a positive set \( A \) contained in \( E \) with
   \( \nu(A) > 0 \).
   (b) State and prove Jordan decomposition theorem.
(OR)

8 State and prove the Radon Nikodym Theorem.

UNIT V

9 (a) State and prove the Caratheodary Extension Theorem.

(b) Let $E$ be a set for which $(\mu \times \nu)(E) = 0$. Then show that for almost all $x$, $\nu(E_x) = 0$.

(OR)

10 State and prove Fubini’s Theorem.

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**UNIT –I**
Inner product spaces, Hilbert Space, Further properties of Inner product spaces, Orthogonal Complements and Direct sums, Orthonormal sets and sequences, Series related to Orthonormal sequences and sets.
(Sections: 3.1 to 3.5 of Chapter 3)

**UNIT – II**
Total orthonormal sets and sequences, Legendre, Hermite and Laguerre polynomials, Representation of functionals on Hilbert Spaces, Hilbert-Adjoint Operator, Self-Adjoint, Unitary and Normal operators.
(Sections: 3.6 to 3.10 of Chapter 3)

**UNIT –III**
Spectral theory in finite dimensional normed spaces, Basic concepts, Spectral properties of Bounded Linear Operators, Further properties of resolvent and Spectrum.
(Sections: 7.1 to 7.4 of Chapter -7)

**UNIT –IV**
Banach Algebras, Further properties of Banach Algebras, Compact linear operators on Normed spaces, Further properties of compact linear operators, Spectral properties of Compact linear operators on Normed spaces.
(Sections 7.6 , 7.7 of Chapter 7 & Sections 8.1 to 8.3 of Chapter -8)

**UNIT –V**
Further Spectral properties of Compact linear operators, Operator equations involving Compact linear operators, Further Theorems of Fredholm type, Fredholm alternative.
(Sections 8.4 to 8.7 of Chapter -8)

**PRESENTED BOOK:** Erwin Kreyszig, *Introductory Functional analysis with Applications*, John Wiley & Sons.

**REFERENCE BOOK:** M. Thamban Nair, *Functional Analysis- A First Course*, PHI
UNIT I

1. a) State and prove Schwarz inequality on an Inner product space.

   b) If X is an inner product space and M is a non empty convex subset of X which is complete, then prove that for any x belongs to X, There exists a unique yє M such that
   \[ \delta = \inf_{y \in M} \| x - y \| = \| x - y \| . \]

   (OR)

2. a) State and prove Bessel inequality.

   b) Show that for any subset \( M \neq \phi \) of a Hilbert space H, M is dense in H if and only if \( M^\perp = \{ 0 \} \).

UNIT II

3. a) In any separable Hilbert space, Prove that every orthonormal set is countable.

   b) State and prove Riesz’s theorem.

   (OR)

4. a) Define Hilbert adjoint operator. Show that the Hilbert adjoint operator \( T^* \) of a bounded linear operator T is also a bounded linear operator with the norm \( \| T^* \| = \| T \| \)

   b) Define a self adjoint operator. Show that a bounded linear operator T: H \( \rightarrow \) H, where H is an Hilbert space, then \( \langle Tx, x \rangle \) is real for all x in H.

UNIT III

5. a) Let Tє \( B(X, X) \), where X is Banach space. If \( \| T \| < 1 \) then Prove that \( (I - T)^{-1} \) exists as a bounded linear operator on the whole space X and \( (1 - T)^{-1} = \sum_{k=0}^{\infty} T^k \).

   b) Prove that the resolvent set \( \rho (T) \) of a bounded linear operator T on a complex banach space X is open and hence show that the spectrum \( \sigma (T) \) is closed.
6. a) Show that the spectrum $\sigma(T)$ of a bounded linear operator $T: X \to X$ on a compact Banach space $X$ is compact and lies in the disk given by $|\lambda| \leq \|T\|$. 

b) Show that the resolvent $R_\lambda$ of $T \in B(X, X)$ satisfies the Hilbert relation 
$$R_\mu - R_\lambda = (\mu - \lambda)R_\mu R_\lambda, \lambda, \mu \in \rho(T).$$

UNIT IV

7. a) Let $X$ and $Y$ be normed spaces and $T: X \to Y$ be a linear operator. Show that $T$ is compact if and only if it maps every bounded sequence $\{x_n\}$ in $X$ on to a sequence $\{Tx_n\}$ in $Y$ which has a convergent subsequence.

b) If $B$ is a totally bounded subset of a metric space $X$, then show that $B$ is separable.

(OR)

8. a) Show that the set of all eigenvalues of a compact linear operator $T$ on a normed linear space $X$ is countable.

b) Let $T: X \to Y$ be a compact linear operator $T$ on a normed space $X$, then prove that for every $\lambda \neq 0$ the range of $T_\lambda = T - \lambda I$ is closed.

UNIT V

9. a) Let $T$ be a compact linear operator on a normed space $X$ and if $T$ has non zero spectral values, then prove that every one of them must be an eigenvalue of $T$.

b) If $T: X \to Y$ is a bounded linear operator on a normed space $X$ and $\lambda \neq 0$, then prove that there exists smallest integer $r$ such that $X = N(T_\lambda^r) \oplus T_\lambda^r(X)$

(OR)

10. Let $T: X \to Y$ be a bounded linear operator on a normed space $X$ and let $\lambda \neq 0$. Then prove that the equations $Tx - \lambda x = 0$ and $T^* f - \lambda f = 0$ have the same number of linearly independent solutions.

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(MAT 403)
IV SEMESTER

MAT 404 A – ALGEBRAIC CODING THEORY

UNIT –I

Introduction to Coding Theory: Introduction, Basic assumptions, Correcting and Detecting error patterns, Information Rate, The Effects of error Correction and Detection, Finding the most likely codeword transmitted, Some basic algebra, Weight and Distance, Maximum likelihood decoding, Reliability of MLD.
(Section 1.1 to 1.10 of Chapter 1)

UNIT – II

Introduction to Coding Theory: Error- detecting Codes, Error – correcting Codes
Linear Codes : Linear Codes, Two important subspaces, Independence, Basis, Dimension, Matrices, Bases for C= <S> and C⊥
(Sections 1.11, 1.12 of chapter 1 & Sections 2.1 to 2.5 of chapter 2).

UNIT – III

Linear Codes: Generating Matrices and Encoding, Parity – Check Matrices, Equivalent Codes, Distance of a Linear Code, Cosets, MLD for Linear Codes, Reliability of IMLD for Linear Codes.
(section 2.6 to 2.12 of chapter 2)

UNIT –IV

Perfect and Related Codes: Some bounds for Code, Perfect Codes, Hamming Codes, Extended Codes, The extended Golay Code, Decoding the extended Golay Code, The Golay code, Reed – Mullar Codes, Fast decoding for RM (1,m).
(Chapter 3)

UNIT –V

Cyclic Linear Codes: Polynomials and Words, Introduction to Cyclic codes, Polynomials encoding and decoding, Finding Cyclic Codes, Dual Cyclic Codes.
(Chapter 4)


M.Sc. DEGREE EXAMINATION
MODEL QUESTION PAPER
Fourth Semester
Mathematics
Paper IV – ALGEBRAIC CODING THEORY

Time: Three hours
Marks: 70

Answer ONE question from each unit.

UNIT I

1. (a) Define CMLD, IMLD. Construct IMLD table for the code C = \{0000,1001,0110,1111\}
   (b) Suppose p=0.90, |M|=3, n=4, and C =\{0000,1010,0111\}. For each v in C, calculate \(\theta_p(C, v)\).

   (OR)

2. (a) Define distance, weight. Suppose we have a BSC with \(\frac{1}{2} < p < 1\). Let \(v_1\) and \(v_2\) be code words, \(w\) a word, each of length n and \(v_1, w\) disagree in \(d_1\) positions and \(v_2, w\) disagree in \(d_2\) positions. Then show that \(\phi_p(v_1, w) \leq \phi_p(v_2, w)\) if and only if \(d_1 \geq d_2\).
   (b) Find the error patterns that corrected by C =\{000,111\}

UNIT II

3. (a) Show that a code C of distance d will at least detect all non zero error patterns of weight less than or equal to (d-1) and there is at least one error pattern of weight d which C will not detect.
   (b) Find the largest linearly independent set from the following sets
      (i) \(S = \{1101, 0111, 1100, 0011\}\)
      (ii) \(S = \{110, 011, 101, 111\}\).

   (OR)

4. (a) For each of the following sets \(S\), find a basis \(B\) for the code \(C=\langle S\rangle\) and a basis for the dual code where \(S =\{111100,000111,101010,010101\}\).
   (b) Find a generator matrix in RREF for the code \(C =\{00000,11100,00111,11011\}\).

UNIT III

5. (a) Find a parity check matrix for the code \(C =\{000000,010101,101010,111111\}\)
   (b) List the Cosets of the linear code C with the generator matrix
6  (a) Construct an SDA for the code \( C = \{0000, 1010, 1101, 0111\} \) assuming IMLD.

(b) If \( H \) is a parity–check matrix for a linear code \( C \) then show that \( C \) has distance \( d \) if and only if any set of \((d-1)\) rows of \( H \) are linearly independent and at least one set of \( d \) rows of \( H \) is linearly dependent.

UNIT IV

7.  (a) State and prove Hamming bound theorem.

(b) Construct an SDA for a Hamming code of length 7 and use it to decode the word 1101011.

8.  (a) Write IMLD for \( C_{24} \).

(b) Using the IMLD for \( C_{24} \) decode the following word \( w = 001001001101, \ 101000101000 \).

UNIT V

9  (a) Show that \( g(x) \) is a generator polynomial for a linear cyclic code \( C \) of length \( n \) if and only if \( g(x) \) divides \((1+x^n)\).

(b) Find a parity check matrix for the linear cyclic code of length 7 with generator \( g(x) = 1 + x + x^2 + x^4 \).

(OR)

10  (a) Find the generator polynomial for all linear cyclic codes of length \( n=4 \).

(b) Find all idempotent polynomials mod \((1+x^7)\).

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(MAT 404 A)
KRISHNA UNIVERSITY
M.SC  MATHEMATICS SYLLABUS
(with effect from the batch admitted during 2012-13)

IV SEMESTER

MAT 404 B – FUZZY SETS AND APPLICATIONS

UNIT-1
From Classical (Crisp) sets to Fuzzy sets: Introduction, Crisp Sets: An overview, Fuzzy set: Basic types, Fuzzy sets: Basic Concepts, Characteristics and significance of the paradigm shift
Fuzzy sets versus Crisp sets: Additional Properties of α-cuts, Representations of Fuzzy sets, Extension principle for Fuzzy sets (Chapters 1 and 2).

UNIT – II

UNIT- III

UNIT-IV

UNIT-V
Fuzzy Logic: Classical Logic: an Over View, Multivalued Logics, Fuzzy Propositions, Fuzzy Quantifiers, Linguistic Hedges, Inference from conditional Fuzzy Propositions, Inference from conditional and qualified propositions, Inference from Quantified propositions. (Chapter 8)


1. (a) Let $A$, $B$ be fuzzy sets defined on a universal set $X$. Prove that $|A| + |B| = |A ∪ B| + |A ∩ B|$.

(b) Let $A, B$ be two fuzzy sets of a universal set $X$. The difference of $A$ and $B$ is defined by $A - B = A ∩ B^c$; and the symmetric difference of $A$ and $B$ is defined by $A Δ B = (A - B) ∪ (B - A)$. Prove that (i) $(A Δ B) Δ C = A Δ (B Δ C)$.

(ii) $A Δ B Δ C = (A ∩ B^c ∩ C) ∪ (A^c ∩ B ∩ C) ∪ (A ∩ B^c ∩ C) ∪ (A ∩ B ∩ C)$.

(OR)

2. (a) State and prove second decomposition theorem.

(b) Let $f : X → Y$ be an arbitrary using function. Then prove that for any $A ∈ f(X)$ and all $α ∈ [0, 1]$, the following properties of $f$ fuzzified by the extension principle hold

(i) $\alpha^+ f(A)_{\leq} f(α^+ A)$

(ii) $\alpha^+ f(A)_{\geq} f(α^+ A)$

UNIT II

3. (a) State axioms of fuzzy complements. Prove that a function $C : [0, 1] → [0, 1]$ satisfies axioms $C_2$ and $C_4$. Then $C$ also satisfy axioms $C_1$ and $C_3$. Moreover, $C$ must be a bijective function.

(b) State and prove every fuzzy complement has at most one equilibrium.

(OR)

4. (a) State and prove second characterization theorem of fuzzy complement.

(b) Prove that the standard fuzzy union is the only the idempotent $t$-norm.

UNIT III

5. Let $*E [+, -, /]$ and let $A, B$ denote continuous fuzzy numbers. Then prove that the fuzzy set
\((A \ast B)(z) = \sup_{z = x \ast y} \min[A(x), B(y)]\) for all \(z \in R\) is a continuous fuzzy number.

**OR**

6. (a) Let \(A\) and \(B\) be fuzzy numbers. Is \(A - B\) a solution of \(\lambda + B = A\)? Justify your answer.

(b) For any fuzzy numbers \(A, B, C\) prove the following:

(i) \(\text{MIN } [A, \text{MAX } (A, B)] = A\)

(ii) \(\text{MIN } [A, \text{MIN } [B, C]] = \text{MIN } [\text{MIN } [A, B], C]\).

**UNIT IV**

7. (a) For any fuzzy relation \(R\) on \(X^2\) prove that the fuzzy relation \(R_{T(i)} = \bigcup_{n=i}^{\infty} R^{(n)}\) is the \(i\)-transitive closure of \(R\).

(b) Let \(R(X, X)\) and \(R(Y, Y)\) be a fuzzy relation defined on the sets \(X=\{a, b, c, d\}\) and \(Y=\{\alpha, \beta, \gamma\}\) respectively are given below:

\[
\begin{array}{cccc}
 a & b & c & d \\
 a & 0 & .5 & 0 & 0 \\
 b & 0 & 0 & 0 & 0 \\
 c & 1 & 0 & 0 & .5 \\
 d & 0 & .6 & 0 & 0 \\
\end{array}
\quad\begin{array}{ccc}
 \alpha & \beta & \gamma \\
 \alpha & .5 & .9 & 0 \\
 \beta & 1 & 0 & .9 \\
 \gamma & 1 & .9 & 0 \\
\end{array}
\]

Define \(h: X \to Y\) be \(h(a) = h(b) = \alpha, h(c) = \beta, h(\alpha) = \gamma\), prove that \(h: X \to Y\) is a homomorphism.

**OR**

8. Let \(a, b, d \in [0,1]\). Prove the following:

(a) \(i(a, b) \leq d\) if and only if \(W_i(Q, d) \geq b\)

(b) \(W_i(W_i(a, b), b) \geq a\)

(c) \(W_i(i(a, b), d) = W_i(a, W_i(b, d))\)

(d) \(a \leq b \Rightarrow W_i(Q, d) \geq W_i(b, d)\) and \(W_i(d, a) \leq W_i(d, b)\)

**UNIT V**

9. Let a fuzzy proposition of the form \(S(x)\) be given, where \(S\) is the identity function (i.e., \(S\) stands for true), and let a fact be given in the form \(\chi\) is \(A'\) where \(\sup_{x \in A(x)} A'(x) = A'(x_0)\) for all \(a \in [0,1]\) and some \(x_0\) such that \(A(x_0) = a\). Then show that the inference “\(\chi\) is \(B'\)” obtained by the method of truth value restrictions is equal to the one obtained by the generalized modus ponens, provided that we use the same fuzzy implication in both inference methods.

**OR**
10. (a) Let sets of values of variables $x$ and $y$ be $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$, respectively. Assume that a proposition “if $x$ is $A$, then $y$ is $B$” is given, where $A = 0.5/x_1 + 1/x_2 + 0.6/x_3$ and $B = 1/y_1 + 0.4/y_2$. Then given a fact expressed by the proposition “$x$ is $A'$,” where $A' = 0.6/x_1 + 0.9/x_2 + 0.7/x_3$, use the generalized modus ponens to derive a conclusion in the form “$y$ is $B'$.”

(b) Define Unconditional and Unqualified propositions and explain these propositions using an example.

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UNIT-1

The Elementary Theory of Near-Rings:
(a) Fundamental definitions and properties: Near-rings, N-groups, Substructures, Homomorphisms and Ideal-like concepts, Annihilators, Generated objects.
(b) Constructions: Products, Direct sums & Subdirect products.
(c) Embeddings: Embedding in $M(\Gamma)$, More beds.

UNIT-II

Ideal Theory:
(a) Sums: (1) Sums and direct sums (2) Distributive sums.
(b) Chain conditions
(c) Decomposition theorems
(d) Prime ideals (1) Products of subsets (2) Prime ideals (3) Semiprime ideals
(e) Nil and nil potent.

UNIT-III

Elements of the structure theory:
(a) Types of N-groups
(b) Change of the near-ring
(c) Modularity
(d) Quasi-regularity
(e) Idempotents
(f) More on Minimality.

UNIT-IV

Primitive Near-Rings:
(a) General (1) Definitions and elementary results (2) The centralizer (3) Independence and density
(b) 0-Primitive near-rings
(c) 1-Primitive near-rings
(d) 2-Primitive near-rings
(1) 2-Primitive near-rings
(2) 2-primitive near-rings with identity.

UNIT-V

Radical Theory: (a) Jacobson-type radicals: Common Theory, (1) Definitions and Characterizations of the radicals (2) Radicals of related near-rings(3) Semi simplicity.
b) Jacobson – type radicals: Special Theory c) The Nil Radical d) The Prime Radical

M.Sc. DEGREE EXAMINATION
MODEL QUESTION PAPER
Fourth Semester
Mathematics
Paper IV – NEAR RINGS

Time: Three hours
Answer ONE question from each unit.
All questions carry equal marks.

Maximum: 70 Marks

UNIT – I

1 (a) (i) If \( I \triangleleft N \), then show that the canonical mapping \( \pi : N \rightarrow \frac{N}{I} \) is a near ring epimorphism.

(ii) If \( h : N \rightarrow N^1 \) is an epimorphism, then show that \( \ker h \triangleleft N \) and \( \frac{N}{\ker h} \cong N^1 \).

(b) Show that each Near ring is isomorphic to a sub direct product of subdirectly irreducible Near rings.

(OR)

2 (a) Show that \( B \subset \Gamma \) is a base for \( \Gamma \) if and only if the inclusion mapping \( i : B \rightarrow \Gamma \) can be extended to an \( N \)-isomorphism \( \phi \rightarrow \Gamma \), where \( \phi \) is the free \( N \)-group on \( B \).

(b) If \( N \) is a zero symmetric Near ring, then show that the left cancellation law implies the right one.

UNIT – II

3 (a) If \( I \triangleleft N \) and \( N \) has the DCCI then show that \( \frac{N}{I} \) also satisfies DCCI.

(b) If \( I \triangleleft N \) and \( I \) is a direct summand then show that \( N \) has DCCN if and only if \( \frac{N}{I} \) and \( I \) have the DCCN.

(OR)

4 (a) Let \( N \) be a Near ring with DCCI. Then show that \( N \) is the finite direct sum of indecomposable ideals.

(b) Let \( I \triangleleft N \). Then show that \( \frac{N}{I} \) has no nilpotent ideals if and only if \( I \) is semi prime.
UNIT –III

5(a) If \(N = N_0\) has the DCCN and \(N \leq N\) is monogenic (by \(m_0\)) , then show that \(M\) contains a right identity and \((0 : m_0)_{M'} = \{0\}\). 

(b) Let \(\Gamma\) be an \(N\) – group and \(\Delta\) be a subset of \(\Gamma\). Then show that \(N\Gamma\) is faithful if and only if \(N_0\Gamma\) and \(N_\Delta\Gamma\) are faithful.

(OR)

6(a) Let \(L\) be modular by \(e\). Then show that \(L \supseteq L^*: N \supseteq L^*\) and this is the greatest ideal of \(N\) contained in \(L\).

(b) If \(N\) is a direct sum of two modular left ideals , then show that \(N\) contains a right identity.

(c) show that each nil ideal \(I\) of a near ring \(N\) is quasi regular.

UNIT –IV

7(a) If \(N\) is commutative and \(\nu\) – primitive , then show that \(N\) is a field.

(b) Show that each \(0\)-primitive ideal is a prime ideal \(\neq N\).

(OR)

8(a) Show that every maximal modular ideal \(I\) of a zero symmetric Near ring \(N\) is a \(0\)-primitive one.

(b) Let \(N\) be zero symmetric Near- Ring with DCCN, \(N\) contains a left identity, \(I \leq N\) and \(N \neq \{0\}\). Then show that \(N\) \(1\)-primitive if and only if \(N\) is a \(2\)-primitive if and only if \(N\) is simple.

UNIT –V

9 Let \(N\) have the DCCI. Then show that \(N\) is \(\nu\) – semi simple if and only if \(N\) is isomorphic to a subdirect product of finitely many \(\nu\) – primitive near rings with DCCI.

(OR)

10(a) Show that \(\mathcal{N}(N)\) is the greatest nil ideal of \(N\).

(b) Show that \(\mathcal{P}(N)\) is a nil ideal and contains the sum of all nilpotent ideals.

(c) Show that each \(\mathcal{P}\)- semi simple Near ring has no non zero nilpotent ideals.

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(MAT 404 C)
UNIT-I
Turing Machines and Linear Bounded Automata: Turing Machine Model. Representation of Turing Machines, Language Acceptability by Turing machines, Design of Turing Machines, Universal Turing machines and Other Modifications. (Sections 7.1 to 7.5 of [1])

UNIT-II
Turing Machines and Linear Bounded Automata: The Model of Linear Bounded Automaton, Turing Machines and Type 0 Grammars, Linear Bounded Automata and Languages, Halting Problem of Turing Machines, NP-Completeness. (Sections 7.6 to 7.10 of [1])

UNIT-III
LR(k) Grammars: LR(k) Grammars, Properties of LR(k) Grammars, Closure properties of Languages. (Sections 8.1 to 8.3 of [1])

UNIT-IV
Computability: Introduction and Basic Concepts, Primitive Recursive Functions, Recursive Functions, Partial Recursive Functions and Turing Machines. (Sections 9.1 to 9.4 of [1])

UNIT-V
Propositions and Predicates: Propositions (Or Statements), Normal Forms of Well-formed Formulas, Rules of Inference for Propositional Calculus (Statement Calculus), Predicate Calculus, Rules of Inference for predicate Calculus. (Sections 10.1 to 10.5 of [1])


UNIT I

1. (a) Define a turing machine and the language acceptable by a turing machine.

(b) Design a turing machine M to recognise the language \( \{2^n 3^n / n \geq 1 \} \).

(OR)

2. (a) Construct a turing machine that can accept the string over \{0, 1\} containing even number of 1’s.

(b) Design a turing machine to recognize the language \( \{a^n b^n c^n / m,n \geq 1 \} \).

UNIT II

3. (a) Prove that the Halting problem of Turing machine over \( \$ = \{0, 1\} \) is unsolvable.

(b) Does PCP with two lists \( x = (b, bab^3, ba) \) and \( y = (b^3, ba, a) \) have a solution? Explain.

(OR)

4. (a) Describe the construction of a grammar corresponding to a turing machine.

(b) Prove that the empty HP of TM is unsolvable.

UNIT III

5. (a) Show that the grammar \( S \rightarrow aAb, A \rightarrow aAb / a \) is LR(1). Is it LR(0) verify.

(b) Prove that every LR(K) grammar G is unambiguous.

(OR)

6. (a) Let G be a grammar \( S \rightarrow aA, A \rightarrow Abb \). Show that G is an LR(0) grammar.

(b) Show that grammar \( S \rightarrow aA, A \rightarrow 1A1, A \rightarrow 1 \) is not an LR(0).

UNIT IV

7. Define primitive recursive function over N and show that the following functions

(a) \( f(x+y) = x+y \)
(b) \( f(x, Y) = x * y \)

(c) \( f(x, Y) = x^Y \) are primitive recursive.

(OR)

8 (a) Show that the function \( f(x) = x - y \) is partial recursive over \( N \).

(b) Explain the construction of Turing machine for computing the projection \( u_i^m \).

UNIT V

9 (a) Define a tautology. Prove that \( \odot P \Rightarrow Q \land (q \Rightarrow r) \Rightarrow (P \Rightarrow R) \) is a tautology.

(b) Obtain the Principle disjunctive normal form of \( \alpha = (\neg P \lor \neg Q) \Rightarrow (\neg P \land R). \)

(OR)

10(a) Show that \( (\neg P \Rightarrow (\neg P \Rightarrow (\neg P \land )) \) \( \equiv P \lor Q \)

(b) Show that the following argument is valid

All men are mortal

Socrates is a man

Therefore Socrates is mortal.

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(MAT 405 A)
UNIT –I
Duality theory and its Ramifications: Alternative formulations of linear programming problems, Dual linear programming problems, Fundamental properties of dual problems, Other formulations of dual problems, Unbounded solution in the primal, The dual simplex algorithm –an example. Post optimality problems, Changing the price vector, Changing the requirements vector, Adding variables or constraints (Sections 8.1 to 8.7 & 8.10 of Chapter 8 of [1]).

UNIT –II

UNIT –III
Game theory: Game theory and Linear programming, Introduction, Reduction of a game to a linear programming problem, Conversion of a linear programming problem to a game problem. (Sections 11.12 to 11.14 of Chapter 11 of [1] and Section 10.3 of Chapter10 of [2] )
Integer programming: Introduction, Gomory’s cut, Balas Implicit Enumeration technique. (Sections 7.1,7.2 and 7.4 of Chapter 7 of [2]).

UNIT IV
Job Sequencing: Introduction, Classification, Notations and Terminologies, Assumptions, Sequencing Problems: Sequence for n jobs through two machines, Sequence for n jobs through three machines, Sequence for 2 jobs through m machines, Sequence for n jobs through m machines (Sections 12.1 to 12.5 of chapter 12 of [3])

UNIT V

PRESCRIBED BOOKS:
UNIT I

1. (a) State and prove fundamental theorem of duality.

(b) Use duality to solve the following L.P.P

max \( z = 2x_1 + x_2 \)

\( \text{sub: } x_1 + 2x_2 \leq 10 \)

\( x_1 + x_2 \leq 6 \)

\( x_1 - x_2 \leq 2 \)

\( x_1 - 2x_2 \leq 1 \)

\( x_1, x_2 \geq 0 \)

(OR)

2. (a) Explain dual simplex algorithm.

(b) Solve the following L.P.P by dual simplex method

max \( z = -3x_1 - x_2 \)

\( \text{sub: } x_1 + x_2 \geq 1 \)

\( 2x_1 + 3x_2 \geq 2 \)

\( x_1, x_2 \geq 0 \)

UNIT II

3. (a) Explain the revised simplex method standard form – I

(b) Use revised simplex method to solve the following

max \( z = 2x_1 + x_2 \)

\( \text{sub: } 3x_1 + 4x_2 \leq 6 \)

\( 6x_1 + x_2 \leq 3 \)

\( x_1, x_2 \geq 0 \)

(OR)

4. (a) Explain the computational procedure of revised simplex method standard form – II

(b) Use revised simplex method to solve the following
\[
\min z = x_1 + x_2 \\
\text{subject to: } x_1 + 2x_2 \geq 7 \\
4x_1 + x_2 \geq 6 \\
x_1, x_2 \geq 0
\]

UNIT III

5 (a) Explain the solution of 2x2 rectangular games.

(b) Solve the following game graphically

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Players} & \text{B}_1 & \text{B}_2 & \text{B}_3 & \text{B}_4 \\
\hline
\text{A}_1 & 2 & 1 & 0 & -2 \\
\text{A}_2 & 1 & 0 & 3 & 2 \\
\hline
\end{array}
\]

(OR)

UNIT IV

6. (a) Explain all integer cutting plane algorithm.

(b) Solve the following IPP

\[
\max z = x_1 + x_2 \\
\text{subject to: } 3x_1 + 2x_2 \leq 5 \\
x_2 \leq 2 \\
x_1, x_2 \geq 0
\]

\[x_1, x_2 \text{ are integers}\]

7. (a) Discuss Johnson’s procedure for determining an optimal sequence for processing n jobs through two machines.

(b) Five jobs are performed first on machine X and then on machine Y. Then time taken in hours by each job on each machine is given below:

<table>
<thead>
<tr>
<th>Jobs</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time on machine X</td>
<td>12</td>
<td>4</td>
<td>20</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>Time on machine Y</td>
<td>6</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

Determine the optimum sequence of jobs that minimizes the total elapsed time to complete the jobs. Also compute the idle time.

(OR)

8. (a) Explain how to process n jobs through m machines.

(b) Find the optimal sequence for the following problem to minimize time and also obtain elapsed time:
UNIT V


(b) BST Roadlines has 4 types of packages A, B, C, and D to be carried in their parcel van. The bulk density of each package is different. As per the company’s rules, the packages fall under different categories of freight classification, and therefore, the revenue for each package are available:

<table>
<thead>
<tr>
<th>Type of package</th>
<th>Wt. per unit(kg)</th>
<th>Expected unit revenue(Rs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2000</td>
<td>200</td>
</tr>
<tr>
<td>B</td>
<td>4000</td>
<td>425</td>
</tr>
<tr>
<td>C</td>
<td>5000</td>
<td>550</td>
</tr>
<tr>
<td>D</td>
<td>3000</td>
<td>350</td>
</tr>
</tbody>
</table>

Determine the number of units of each package that would maximize the revenue, given that the capacity of van is limited to 10,000 kg.

(OR)

10. (a) Show how to solve a linear programming problem by dynamic programming approach.

(b) Solve the following linear programming problem by applying dynamic programming procedure.

\[ \text{Max} Z = 2x_1 + 5x_2 \]

\[ \text{subject to:} \ 2x_1 + x_2 \leq 43 \]
\[ 2x_2 \leq 46 \]
\[ x_1, x_2 \geq 0 \]

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(MAT 405 B)