There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I
Laplace Transforms: Existence of Laplace transform, Functions of exponential, Shifting theorems, Scale property, Laplace transforms of derivatives, Initial and final value theorems, Laplace transforms of integrals, multiplication by \( t^n \) and division by \( t \), Laplace transform of periodic and some special function. (Chapter 1 of Ref.(1)).

UNIT-II
Inverse Laplace transforms: shifting theorems and scale property of inverse Laplace transforms, Use of partial fractions, Inverse Laplace transforms to derivatives and integrals, Multiplication and division by powers of \( p \), convolution theorem, Heaviside’s expansion theorem, complex inversion formulae. (Chapter 2 of Ref.(1))

UNIT-III
Applications to differential equations: Solutions of ordinary differential equations (ODE) with constant and variable coefficients, solutions of simultaneous ODEs and partial differential equations. Applications to electrical circuits and mechanics, Applications to simple Integral equations, Applications of Laplace transforms in initial and Boundary value problems. (Chapters 3, 4 to 5 of Ref.(1))

UNIT-IV
Fourier Transforms: Fourier Sine and Cosine transforms, Shifting property, convolution theorem, Parseval’s identity, Finite Fourier transforms: Finite Fourier Sine and Cosine transforms, Multiple finite Fourier transforms, operational properties of finite Fourier Sine and Cosine transforms, Applications of Fourier Transforms in initial and boundary value problems. (Chapter 6 to 8 of Ref.(1))

UNIT-V
CALCULUS OF VARIATIONS: Maxima and Minima of functions and Functionals, Euler's equations and standard problems of calculus of variations, Theory of Small Vibrations and Vibrating String. (Chapter 6 of Ref(2))

REFERENCES:

1. Integral transforms by Vasistha and Gupta, Krishna prakashan, Meerut.(2000)

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

**UNIT - I**

Generalized coordinates, Velocities, Forces, Holonomic and non-holonomic systems, conservative and non-conservative systems. Lagrange’s equations of motion of holonomic (non-holonomic) and conservative (non-conservative) systems. Theory of small oscillations of conservative holonomical dynamical systems. *(Sections 10.2 to 10.12 of Ref.(1) and for problems Ref.(2))*

**UNIT - II**

Hamilton’s principle for holonomic(non-holonomic) and conservative(non-conservative) systems from Lagrange’s equations of motion, Cyclic coordinates, Conservation theorems, Routh’s procedure and Hamilton’s equation of motion from variational principle and from modified Hamilton’s Principle, Principle of least action for holonomic and non-holonomic systems. *(Sections 2.1, 2.3, 2.4, 2.6, 8.1 to 8.3, 8.5 and 8.6 of Ref.(3))*

**UNIT - III**

Canonical Transformations, Jacobi’s theorem, Types of canonical transformation equations, Examples of C.T. Solution of Harmonic oscillator problem using canonical transformation, Symplectic approach to a canonical transformation, Infinitesimal canonical transformation, canonical transformations form a group, Exact differential condition. *(Sections 9.1 to 9.4 of Chapter 9 of Ref.(3), relevant articles in Ref.(4) for UNIT-III & IV)*

**UNIT - IV**

Bilinear invariant condition. Poisson and Lagrange brackets and invariance of them under C.T, Relation between Lagrange and Poisson brackets. Conditions for C.T. in terms of Lagrange and Poisson brackets. *(Section 9.4(Pages 399, 400, 402, 403), section 9.5(Pages 405, 406, 407) of Ref.(2), sections 10.1,10.2 and 10.3 of Ref.(3))*

**References**

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I

**Metric spaces**: Metric spaces, Complete metric spaces, Compactness and continuity.
*(Chapter I of Ref. (1))*

UNIT-II

**Linear metric Spaces**: Vector spaces, Linear metric spaces, Normed linear spaces.
*(Chapter II of Ref. (1))*

UNIT-III

**Basic theorems on normed linear spaces**: Bounded linear transformations, Hahn-Banach theorem, Open mapping theorem, Banach–Steinhaus theorem.
*(Chapter III of Ref. (1))*

UNIT-IV

**Hilbert Spaces**: Inner product spaces, Orthonormal sets, Riesz Representation theorem, Bounded linear operators on Hilbert spaces.
*(Chapter V of Ref. (1))*

UNIT-V

**Fixed point Theory**: The Contraction mapping theorem and its application, Brouwer’s fixed point theorem without proof and its application, Schauder’s fixed point theorem without proof and some related results.
*(Chapter IX of Ref. (1))*

**REFERENCE**:
1. Functional Analysis with Applications by B. Choudhary and Sudarsan Nanda, Published by wiley Eastern Limited, New Delhi.
There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I

**Kinematics of fluids:** Real and ideal fluids, Velocity of fluid at a point, Streamlines and path lines, Velocity potential, Vorticity vector, Local and particle rates of change, Equation of continuity, Acceleration of a fluid, conditions at a rigid boundary. *(Sections 2.1 to 2.10 of Chapter 2 of Ref(1)).*

UNIT-II

General analysis of fluid motion. **Equations of motion of a fluid:** Pressure at a point in a fluid at rest and in a moving fluid, conditions at a boundary of two in viscid immiscible fluids, Euler’s equations of motion, Bernoulli’s equation, discussion of steady motion under conservative body forces. *(Section 2.11 of Chapter 2 and Sections 3.1 to 3.7 of Chapter 3 of Ref.(1)).*

UNIT-III

Potential theorems, Flows involving axial symmetry, Impulsive motion, Kelvin's circulation theorem. **Two dimensional flows:** Meaning of two dimensional flow, Stream function, Complex potential for two dimensional irrotational incompressible flows, Complex velocity potentials for standard two dimensional flows. *(Sections 3.8,3.9,3.11,3.12 of Chapter 3 and Sections 5.1 to 5.5 of Chapter 5 of Ref.(1)).*

UNIT-IV

Uniform Stream, Line source, Line sinks, Line doublets, Line Vortices; Two dimensional image systems, Milne Thomson Circle theorem, applications of Circle theorem, Blasius theorem. *(Sections 5.5.1. to 5.9 of Chapter 5 of Ref.(1)).*

UNIT-V

**Three dimensional flows:** Introduction, Three Dimensional sources, sinks and doublets, Images in rigid infinite plane, Images in solid spheres, Weiss’s sphere theorem, Axi symmetric flows, Stokes stream function, some special forms of the stream function for Axi-symmetric irrotational motions. *(Chapter 4 of Ref.(1)).*

**REFERENCES:**

Max. Marks: 70  
Time: Three hours

There are five questions and each carrying 14 marks. One has to answer one from each unit with an internal choice of selecting one from given two.

UNIT-I

Linear Programming problem—mathematical formulation, example of linear programming. Graphical examples, Basic solutions of a system of linear equations. Convex sets, Extreme points, Slack and Surplus variables. Reduction of a feasible solution to a basic feasible solution to a basic feasible solution. Improving a basic feasible solution. Unbounded solutions, optimality conditions, alternative optima. Extreme points and basic feasible solutions. (Sections 1.1 to 1.7 of Chapter 1, sections 2.16, 2.20 of chapter 2 and chapter 3 of Ref. (1)).

UNIT-II

The simplex method, selection of vector to enter the basis, intial basic feasible solution—artificial variables, inconsistency and reducny. Artificial basic of techniques. (Chapter 4 (excluding sections 4.3 & 4.4) Sections 5.1 to 5.3 and 5.8 of Chapter 5 of Ref (1)).

UNIT-III

Determination all optimal solutions, Unrestricted variables. Examples of cycling, Resolution of the Degeneracy problems, Charne’s perturbation method. (Sections 6.1 to 6.5 and 6.10 of Chapter 6 of Ref. (1)).

UNIT-IV

Dual linear programming problems (Chapter 8 of Ref. (1)).

UNIT-V

General Transportation problem (T.P.P), Transportation Table, Solution of a T.P.P, Finding intial basic feasible solution, Test optimality, Degeneracy in T.P.P., Modi method. Assignment Promblems. (Sections 10.1 to 10.3, 10.8 to 10.12 of Chapter 10, Sections 11.1 to 11.3 of chapter 11 of Ref. (2)).

REFERENCE:
M.Sc.(Previous)DEGREE EXAMINATION,  
Model paper  
Third Semester  
Applied Mathematics  
Paper I – Techniques of Applied Mathematics  
Time : Three hours     Maximum : 70 marks  
Answer ALL questions. Each question carries 14 marks

Unit-I

1(a) State and prove First and second shifting theorems.

(b) Find the Laplace Transform of (i) \( \frac{\cos(at) - \cos(bt)}{t} \) and (ii) \( J_0(t) \).

(OR)

2(a) Find the Laplace transform of nth order derivative of \( F(t) \).

(b) State and prove initial value theorem.

Unit-II

3(a) Find the Laplace inverse transformation of \( \frac{3p^2 + 2}{4p^2 + 12p + 9} \).

(b) Show that \( \int_0^\infty \cos x^2 \, dx = \frac{1}{2} \sqrt{\pi/2} \).

(OR)

4(a) State and prove convolution theorem for Laplace transforms.

(b) Apply convolution theorem to prove that

\[
\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} \, dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, m > 0, n > 0.
\]

Hence deduce that

\[
\frac{\pi}{2} \int_0^1 x^{2m-1}(1-x)^{n-1} \, dx = \frac{1}{2} \beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}.
\]
5(a) Solve \((D^2 + 9)y = \cos 2t\), if \(y(0) = 1, y\left(\frac{\pi}{2}\right) = -1\).

(b) An alternating E.M.F. \(E \cdot \sin \omega t\) is applied to an inductance \(L\) and a capacitance \(C\) in series. Show that the current in the circuit is

\[
\frac{E\omega}{(n^2 - \omega^2)} (\cos \omega t - \sin \omega t), \text{where } n^2 = \frac{1}{LC}.
\]

(OR)

6(a) Solve by Laplace transform method

\[
\frac{dy}{dt} + 2y + \int_0^t y \, dy = \sin t, \ y(0) = 1.
\]

(b) Solve the boundary value problem

\[
\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, x > 0, t > 0
\]

Where \(u(x, 0) = 0, u_t(x, 0) = 0, x > 0, u(0, t) = F(t)\) and \(\lim_{n \to \infty} u(x, t) = 0, \ \forall t \geq 0\).

Unit - IV

7(a) Find the Fourier transform of

\[
f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}
\]

And hence evaluate \(\int_0^\infty \sin \frac{x}{x} \, dx\).

(b) Obtain relationship between Fourier and Laplace transforms.

(OR)

8 Use the method of Fourier transform determine the displacement \(y(x, t)\) of an infinite string, given that the string is initially at rest and that the initial displacement is \(f(x), -\infty < x < \infty\). Show that the solution can also be put in the form \(y(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)]\).
Unit - V

9(a) Obtain a Necessary functional

\[ J(y(x)) = \int_{x_0}^{x_1} F(x, y(x), y'(x)) dx \]

satisfying \( y(x_0) = y_0 \) and \( y(x_1) = y_1 \)

(b) Find the extremal of the functional

\[ J(y(x)) = \int_{x_0}^{x_1} \sqrt{1 + (y'(x))^2} \, dy \]

(OR)

10(a) Determine the differential equation satisfied by the free vibrations of a string.

(d) Find the plane curve of fixed perimeter area.
M.Sc.(Previous) DEGREE EXAMINATION,

**Model paper**

Third Semester

**Applied Mathematics**

**Paper II – HIGHER MECHANICS**

**Time:** Three hours  
**Maximum:** 70 marks

*Answer ALL questions. Each question carries 14 marks*

**UNIT-I**

1. (a) Derive Lagrange’s equations of motion for holonomic non-conservative dynamical system.
   (b) A uniform rod of length $2a$ which has one end attached to a fixed point by a light inextensible string of length $125a$ is performing small oscillations in a vertical plane about its position of equilibrium. Show that the period of its principal oscillations are $2\pi \sqrt{\frac{5a}{3g}}$ and $\pi \sqrt{\frac{a}{3g}}$.

(OR)

2. (a) Derive Lagrange’s equations of motion of a body under impulsive forces.
   (b) A homogeneous rod $OA$ of mass $m_1$ and length $2a$ is freely hinged at $O$ to a fixed point; at its other end is freely attached another homogeneous rod $AB$ of mass $m_2$ and length $2b$; the system moves under gravity; find equations to determine motion.

**UNIT-II**

3. (a) Derive Hamilton’s principle for holonomic conservative system.
   (b) Write down Hamilton’s equations of motion for a simple pendulum.

(OR)

4. (a) Derive the principle of least action for holonomic dynamical system.
   (b) Obtain modified Hamilton’s principle.

**UNIT-III**

5. (a) Find the solution of Harmonic oscillation problem using canonical transformation.
   (b) Show that the set of all canonical transformations forms a group under the composition of transformations.

(OR)

6. (a) Derive exact differential condition for a transformation to be a canonical.
   (b) Show that the transformation $Q = \log\left(\frac{1}{q} \sin p\right)$, $P = q \cot p$ is canonical. Also find a generating function.

**UNIT-IV**

7. (a) Define Poisson bracket. Show that the Poisson brackets are invariant under canonical transformation.
   (b) Obtain relation between Poisson and Lagrange brackets.

(OR)
8 (a) Derive bilinear invariant condition for a transformation to be canonical.
(b) Show that the transformation $Q = (2q)^{1/2} k^{-1/2} \cos p$ and $P = (2q)^{1/2} k^{1/2} \sin p$ is a canonical transformation using the bilinear invariant condition.

UNIT-V

9 (a) State and prove Jacobi Identity.
(b) Derive Hamilton-Jacobi equation for Hamilton’s principal function.

(OR)

10 (a) State and prove Poisson theorem.
(b) State and prove Poincare’s theorem.
M.Sc.(Previous) DEGREE EXAMINATION,

Model paper
Third Semester
Applied Mathematics
Paper III – FUNCTIONAL ANALYSIS

Time : Three hours     Maximum : 70 marks
Answer ALL questions. Each question carries 14 marks

Unit-I

1(a) Let (X,d) be a metric space. Prove that the following are equivalent
    (i) X is compact (ii) X is Frechet compact (iii) X is sequentially compact.
(b) Let f be a continuous mapping on a compact metric space (X,d) into a metric space (Y,d), then show that f is uniformly continuous.

(OR)

2 (a) State and prove Baire’s category theorem.
(b) State and prove Ascoli’s theorem

Unit-II

3 (a) Show that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.
(b) State and prove Riesz Lemma.

(OR)

4 (a) Let M be a closed linear subspace of a normed linear space X. Then show that the quotient space X/M is a normed linear space with the norm
    \[ \|x + M\| = \inf \{\|x + m\| : m \in M\}. \]
    Further, if X is a Banach space, show that X/M is a Banach space.
(b) If X is a normed linear space and \{x_n\} is a Cauchy sequence in X then show that \{\|x_n\|\} converges.

Unit-III

5 (a) State and prove Han-Banach theorem for real linear space.
(b) State and prove Banach-Steinhaus theorem.

(OR)

6. State Open mapping theorem and give its proof completely.

Unit-IV

7 (a) State and prove Riesz representation Theorem.

(b) If T_1 and T_2 are normal operators such that each commutes with the adjoint of the other. Show that T_1 + T_2 and T_1 T_2 are normal.

(OR)

8 (a) If K is a non-empty closed and convex set in a Hilbert Space show that there exists a unique vector in K of smallest norm.
(b) Suppose $f \in L_2[-\pi, \pi]$, show that 
\[
\int_{-\pi}^{\pi} f(t) \cos nt \, dt \to 0, \quad \text{as } n \to \infty
\]

\textbf{Unit-V}

9 (a) State and prove Banach Contraction theorem.
(b) State and prove Markov-Kakutani theorem.

(OR)

10 (a) State and prove Picard’s theorem.
(b) State and prove the existence theorem for Volterra integral equation.
M.Sc.(Previous) DEGREE EXAMINATION,  
Model paper  
Third Semester  
Applied Mathematics  
Paper IV – FLUID MECHANICS  
Time : Three hours     Maximum : 70 marks  
Answer ALL questions. Each question carries 14 marks

Unit-I

1. (a) Derive the equation of continuity in spherical polar co-ordinates.  
(b) Show that \( \frac{x^2}{a^2} \tan^2 t + \frac{y^2}{b^2} \cot^2 t = 1 \) is a possible form for the bounding surface of a fluid and find an expression for the normal velocity. 

(OR)

2. (a) Test whether the motion specified by \( \vec{q} = \frac{-y\vec{j} + x\vec{j}}{x^2 + y^2} \) is a possible motion for an incompressible fluid. If so determine the equations of stream lines.  
(b) At the point in an incompressible fluid having spherical polar coordinates \((r, \phi, \psi)\), the velocity components are \((2Mr^{-3} \cos \phi, Mr^{-3} \sin \phi, 0)\), where \(M\) is a constant. Show that the velocity is of the potential kind. Find the velocity potential.

Unit-II

3. Discuss the general analysis of fluid motion.  
(OR)

4. (a) Derive the Euler’s Equations of motion for an inviscid incompressible fluid flow.  
(b) Show that at any point of a moving inviscid fluid the Pressure \(P\) is the same in all directions.

Unit-III

5. (a) State and prove Kelvin’s Circulation theorem.  
(b) Discuss the flow for which \(W = z^2\).  

(OR)

6. Discuss completely, with the usual notation, the motion of a sphere moving with a constant velocity in liquid which is otherwise at rest.

Unit-IV

7. (a) State and prove Milne-Thomson circle theorem.
(b) Describe the irrotational motion of an incompressible fluid for which the complex potential is \( W = ik \log z \). and also obtain image of a line source in a circular cylinder.

(OR)

8 (a) State and prove Blasius Theorem.
(b) Find the force exerted by the fluid on an infinite circular cylinder in uniform stream with circulation in the usual notation.

Unit-V

9 (a) State and prove Weiss’s sphere theorem.
(b) Using the above theorem obtain the image system of a point source in a solid sphere.

(OR)

10 (a) Obtain Stokes stream function for (i) a uniform line source along the axis.
(ii) a doublet along the axis.
(b) Doublets of strength \( \mu_1 \), \( \mu_2 \) are situated at \( A_1(0,0,c_1) \) and \( A_2(0,0,c_2) \) their axis is being directed towards and away from the origin respectively. Find the condition that there is no transport of fluid over the surface of the sphere \( x^2 + y^2 + z^2 = c_1c_2 \).
1.(a) State and Prove Fundemental theorem of linear programming.

(b) Solve the following L.P by graphical method

Maximize \( Z = x_1 + 3x_2 \)

subject to

\[
3x_1 + 6x_2 \leq 8, \quad 5x_1 + 2x_2 \leq 10, \quad x_1, x_2 \geq 0
\]

(OR)

2.(a) Compute all the basic feasible solutions of the LPP

Maximize \( Z = 2x_1 + 3x_2 + 4x_3 + 7x_4 \)

subject to

\[
2x_1 + 3x_2 - x_3 + 4x_4 = 8
\]

\[
x_1 - 2x_2 + 6x_3 - 7x_4 = -3
\]

and choose that one which maximizes \( Z \).

(b) what is a feasible solution for a LPP? Show that set of all feasible solutions of a LPP forms a Convex Set.

Unit-II

3.(a) Solve the following LPP by Simplex Procedure:

Maximize \( Z = 5x_1 + 3x_2 \)

subject to the constraints  \( 5x_1 + 2x_2 \leq 10, \quad 3x_1 + 8x_2 \leq 12, \quad x_1, x_2 \geq 0 \)

(b) Use two-phae simplex method to Maximize \( Z = 3x_1 + 2x_2 \)
subject to the constraints \( 2x_1 + x_2 \leq 2, \quad 3x_1 + 4x_2 \geq 12, \quad x_1, x_2 \geq 0 \)

(OR)

4.(a) Explain the Simplex algorithm of solving a LPP.

(b) Solve the following LPP by Big-M- method

\[
\text{Minimize } Z = 4x_1 + 3x_2 \\
\text{subject to } \quad 2x_1 + x_2 \geq 10, \quad -3x_1 + 2x_2 \leq 6, \quad x_1 + x_2 \geq 6 \quad \text{and} \quad x_1, x_2 \geq 0.
\]

**Unit-III**

5.(a) What is meant by degeneracy and cycling in the simplex method of solving LPP and explain how you overcome these problems to obtain an optimal solution.

(b) Solve the following LPP using Charene’s Perturbation technique:

Maximize \( Z = 22x_1 + 30x_2 + 25x_3 \),

subject to the \( x_1 + x_2 \leq 100, \quad x_1 + 2x_2 + x_3 \leq 100, \quad 2x_1 + x_2 + x_3 \leq 100, \quad x_1, x_2, x_3 \geq 0 \)

(OR)

6.(a) Explain Charne’s Perturbation technique

(b) Maximize \( Z = 2x_1 + 3x_2 \),

subject to \( -x_1 + 2x_2 \leq 4, \quad x_1 + x_2 + \leq 6, \quad x_1 + 3x_2 + \leq 9, \quad x_1, x_2 \) are unrestricted

**Unit-IV**

7.(a) State and prove complementary slackness theorem for primal-dual problems.

(b) Use dual simplex method to solve the following LPP

Maximize \( Z = x_1 + 2x_2 + 3x_3 \),

subject to the \( x_1 - x_2 + x_3 \geq 4, \quad x_1 + x_2 + 2x_3 \leq 8, \quad x_1 - x_3 \geq 2, \quad x_1, x_2, x_3 \geq 0 \)

(OR)

8.(a) Explain the primal-dual relationships. Show that if either the primal or dual problem
has a finite optimum solution, then the other problem also has finite optimal solution

and the values of the both objective functions are equal.

(b) What is meant by duality in LP? Show that the dual of the dual of a given primal is again

the primal problem.

**Unit-V**

9. (a) State the transportation problem in the format of a LPP. When does it have a unique

solution? Explain.

(b) Determine an optimal solution for the following TPP

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(OR)

10. (a) Find the optimal solution to the following TPP by North-West Corner rule

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(b) Explain the method of solving an assignment problem.